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ANALYTICAL AND NUMERICAL INVESTIGATION OF THE THREE-DIMENSIONAL VISCOUS SHOCK-LAYER ON BLUNT SOLIDS

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Three-dimensional problems of viscous flow around bodies are presently among the most pressing problems of hypersonic aerodynamics in connection with the development of craft which move in the upper atmospheric layers. The use of numerical methods in solving such problems requires great amounts of computer time and internal computer memory. Therefore, development of approximate methods which, while being sufficiently accurate, can be used in engineering practice, is very timely. Many approximate methods have been developed for large Reynolds numbers Re. They are based on the boundary layer theory and require knowledge of the parameters of nonviscous flow at the surface of the solid. However, there are presently no similar methods suitable for solving three-dimensional problems of viscous flow around solids at small and medium Reynolds numbers ($Re \leq 10^3$), where the viscosity is considerable throughout the entire region of perturbed flow and the classical boundary layer theory is inapplicable.

On the basis of an approximate solution of the equations of a three-dimensional hypersonic viscous shock-layer, we have obtained an analytical solution for determining the thermal flux and the friction stress at the lateral surface of blunt solids for small and medium Re numbers with an allowance for the slippage effect and the temperature jump at the surface. For a flow characterized by medium or large Re values, a simple expression has been derived for the thermal flux distribution over the surface; the thermal flux is reduced to its value at the stagnation point. This expression depends only on the geometry of the body in the flow. The present article is a continuation of [1], where a similar problem was solved in the neighborhood of the symmetry plane.

1. Consider the steady-state, three-dimensional hypersonic flow of a viscous gas around a smooth, blunt solid at small and medium Re numbers. The flow is investigated by

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using the model of a thin, viscous shock-layer, similar to the two-layer model proposed in [2] for axisymmetric flow around a solid and generalized in [3] to encompass the case of three-dimensional flow.

Assume that the surface of the streamlined solid is assigned in a Cartesian coordinate system by the equation $z = f(x, y)$, the velocity vector of the oncoming flow v_∞ has the direction of the z axis, the coordinate origin is located at the stagnation point of the flow, and x and y axes lie in the principal curvature planes of the surface at this point. We choose a system of curvilinear nonorthogonal coordinates $\{x^i\}$, bound to the streamlined surface: x^3 is the distance along the normal to the surface, while the Cartesian coordinates of the point of intersection between this normal and the surface $x^1 = x$, $x^2 = y$, $z = f(x^1, x^2)$ are used as the other two coordinates at the surface.

The equations of the three-dimensional, thin viscous shock-layer in the $\{x^i\}$ coordinate system are given by

$$\frac{\partial}{\partial x^\alpha}\left(\rho u^\alpha \sqrt{\frac{g}{g_{(\alpha\alpha)}}}\right) + \frac{\partial}{\partial x^3}\left(\rho u^3 \sqrt{g}\right) = 0,$$

$$\rho\left(\frac{u^\alpha}{\sqrt{g_{(\alpha\alpha)}}}\frac{\partial u^\gamma}{\partial x^\alpha} + u^3\frac{\partial u^\gamma}{\partial x^3}\right) + \rho A^\gamma_{\alpha\beta}u^\alpha u^\beta = -\sqrt{g_{(\gamma\gamma)}}g^{\beta\gamma}\frac{\partial p}{\partial x^\beta} + \frac{\partial}{\partial x^3}\left(\frac{\mu}{\mathrm{Re}}\frac{\partial u^\gamma}{\partial x^3}\right),$$

$$\gamma = 1, 2,$$

$$\rho A^3_{\alpha\beta}u^\alpha u^\beta = -\frac{\partial p}{\partial x^3}, \tag{1.1}$$

$$\rho\frac{u^\alpha}{\sqrt{g_{(\alpha\alpha)}}}\frac{\partial H^*}{\partial x^\alpha} + \rho u^3\frac{\partial H^*}{\partial x^3} = \frac{\partial}{\partial x^3}\left\{\frac{\mu}{\mathrm{Re\,Pr}}\left[\frac{\partial H^*}{\partial x^3} + (\mathrm{Pr}-1)\frac{\partial}{\partial x^3}\left(\frac{g_{\alpha\beta}}{\sqrt{g_{(\alpha\alpha)}g_{(\beta\beta)}}}u^\alpha u^\beta\right)\right]\right\},$$

$$\frac{p}{\rho} = \varepsilon T, \quad H^* = T + \frac{g_{\alpha\beta}}{\sqrt{g_{(\alpha\alpha)}g_{(\beta\beta)}}}u^\alpha u^\beta, \quad \mu = T^\omega,$$

$$\varepsilon = \frac{\gamma-1}{2\gamma}, \quad \mathrm{Re} = \frac{\rho_\infty v_\infty R}{\mu(T_0)}, \quad T_0 = \frac{v_\infty^2}{2c_p},$$

$$g = g_{11}g_{22} - g_{12}^2, \quad g_{\alpha\alpha} = 1 + f'^2_\alpha, \quad g_{12} = f'_1 f'_2, \; f'_\alpha = \frac{\partial f}{\partial x^\alpha}.$$

Here and below, summation with respect to the subscripts within round brackets is not performed, while the Greek subscripts assume the values 1, 2; $\rho_\infty\rho$ is the density, $\rho_\infty v_\infty^2 p$ is the pressure, $\mu\mu(T_0)$ is the viscosity coefficient, $H^* v_\infty^2/2$ is the total enthalpy, TT_0 is the temperature, $u^i v_\infty$ $(i = 1, 2, 3)$ are the components of the velocity vector, R is the characteristic dimension of the solid, represented by one of the principal curvature radii at the stagnation point, Pr is the Prandtl number, and γ is the ratio of the specific heat characteristics; the subscript ∞ denotes the values of the quantities in the unperturbed flow. The coefficient $A_{\alpha\beta}{}^\gamma$ are certain functions of the metric tensor $g_{\alpha\beta}$ given in [4].

The boundary conditions accounting for the slipping rate and the temperature jump are assigned at the solid's surface:

$$x^3 = 0:\; u^\alpha = \frac{2-\theta}{\theta}\sqrt{\frac{\gamma\pi}{\gamma-1}}\frac{\mu}{\rho\,\mathrm{Re}\sqrt{T}}\frac{\partial u^\alpha}{\partial x^3}, \quad u^3 = 0,$$

$$H^* = H^*_w + \frac{2-\alpha}{\alpha}\frac{2\gamma}{\gamma+1}\sqrt{\frac{\gamma\pi}{\gamma-1}}\frac{\mu}{\rho\,\mathrm{Re\,Pr}\sqrt{T}}\frac{\partial H^*}{\partial x^3}. \tag{1.2}$$

The generalized Rankine—Hugoniot conditions are used at the inside boundary of the shock-wave:

$$x^3 = x^3_s:\; \rho\left(u^3 - \frac{u^\alpha}{\sqrt{g_{(\alpha\alpha)}}}\frac{\partial x^3_s}{\partial x^\alpha}\right) = u^3_\infty,$$

$$u^3_\infty(u^\alpha - u^\alpha_\infty) = \frac{\mu}{\mathrm{Re}}\frac{\partial u^\alpha}{\partial x^3}, \quad p = (u^3_\infty)^2 + \frac{p_\infty}{\rho_\infty v_\infty^2}, \tag{1.3}$$

$$u^3_\infty(H^* - H^*_\infty) = \frac{\mu}{\mathrm{Re\,Pr}}\frac{\partial}{\partial x^3}\left[H^* + (\mathrm{Pr}-1)\frac{g_{\alpha\beta}u^\alpha u^\beta}{\sqrt{g_{(\alpha\alpha)}g_{(\beta\beta)}}}\right],$$

where θ is the coefficient of diffusion reflection, and α is the accommodation coefficient (the θ = 1 and α = 1 values were used in calculations); the w and s subscripts correspond to the parameter values at the solid's surface and the inside boundary of the shock-wave.

The components of the friction stress and the thermal flux are calculated by means of the expressions (the primes denote dimensional quantities)

$$\tau^{\alpha'} = \mu' \left(\frac{\partial u^{\alpha'}}{\partial x^{3'}} + \frac{g_{\alpha\beta}}{\sqrt{g_{(\alpha\alpha)} g_{(\beta\beta)}}} \frac{\partial u^{\beta'}}{\partial x^{3'}} \right), \quad \alpha \neq \beta,$$

$$q' = \lambda' \frac{\partial T'}{\partial x^{3'}} + \mu' \frac{g_{\alpha\beta}}{2\sqrt{g_{(\alpha\alpha)} g_{(\beta\beta)}}} \frac{\partial (u^{\alpha'} u^{\beta'})}{\partial x^{3'}}.$$

The Stanton number and the friction coefficient are determined as follows:

$$c_H = q' / [\rho_\infty v_\infty (H_\infty^* - H_w^*)], \quad c_f^\alpha = \tau^{\alpha'} / (\rho_\infty v_\infty^2).$$

2. The system of equations (1.1) has a singularity at the critical point. In order to resolve this singularity, we pass to new dependent variables, $u^\alpha = u_\infty{}^\alpha u_*{}^\alpha$. In the chosen coordinates system, $u_\infty{}^\alpha = f_\alpha' \sqrt{g_{(\alpha\alpha)}}/g$.

Furthermore, we pass to new independent variables of the Dorodnitsyn type:

$$\xi^\alpha = x^\alpha, \quad \zeta = \frac{1}{\Delta} \int_0^{x^3} \rho \, dx^3, \quad \Delta = \int_0^{x_s^3} \rho \, dx^3.$$

We shall solve the equations of a three-dimensional, viscous shock-layer, written in terms of the new variables, by using the integral method of successive approximations, which was first proposed in [5] for solving two-dimensional boundary-layer equations. A similar method was then developed in [6] for solving two-dimensional problems of a hypersonic viscous shock-layer. The equations of momentum and energy are integrated twice with respect to the transverse coordinate while using boundary conditions (1.2) and (1.3). In order to solve the resulting system of integrodifferential equations, we devised an iteration process where each subsequent approximation of the functions to be determined is expressed in terms of integrals of the preceding approximation. In order to ensure that all approximations satisfy the boundary conditions at both the solid and the shock-wave, we introduce at each step of the iteration process additional control functions $\Delta^\alpha(x^1, x^2)$ and $\Delta_H(x^1, x^2)$, for which partial differential equations are obtained. They generally do not have an analytical solution and are, therefore, solved here by using a locally self-similar approximation.

We assign the initial approximation for the components of the velocity vector $u_*{}^\alpha$ and the reduced total enthalpy $G = (H^* - H_w{}^*)/(H_\infty{}^* - H_w{}^*)$ in the form of linear functions with respect to the transverse coordinate $u_*{}^\alpha = a(\zeta + b)$ and $G = c(\zeta + d)$, where a, b, c, and d depend on x^1 and x^2 and are determined from the boundary conditions at the solid and at the shock-wave. Then, in the first approximation of this method, we obtain the analytical solution for the pressure, the velocity components, the friction coefficient, and the Stanton number:

$$p_w = \frac{1 - \beta_* t}{g}, \quad \beta_* = \frac{g\lambda a}{3r\beta}, \quad t = 1 + 3b + b^2, \quad r = \frac{1}{2} + b; \tag{2.1}$$

$$u_*^\alpha = \sqrt{g}\,\alpha_0 \eta^\alpha \Delta^\alpha + \alpha_1 \Delta^{\alpha^2} (T^1(\zeta) - F^\alpha(\zeta)),$$
$$G = \sqrt{g}\,\alpha_0 \eta \Delta_H + \alpha_1 \Pr \Delta_H^2 T^2(\zeta) + \frac{1 - \Pr}{1 - G_w} \varkappa_{\alpha\beta} u_*^\alpha u_*^\beta; \tag{2.2}$$

$$c_f^\alpha = \frac{2}{\sqrt{g}} \eta^\alpha \Delta^\alpha, \quad \eta^\alpha = r - \frac{1}{3} at - R^\alpha; \tag{2.3}$$

$$c_H = \frac{\eta}{\sqrt{g}} \Delta_H, \quad \eta = r - \frac{c}{3}\left[\frac{3}{2}(b + d) + 3bd + 1\right];$$
$$\lambda = \frac{f_{11}'' f_1'^2 + 2 f_{12}'' f_1' f_2' + f_{22}'' f_2'^2}{g^3}, \quad \varkappa_{\alpha\alpha} = (1 + f_\alpha')^2 \frac{f_\alpha'^2}{g^2}, \quad \varkappa_{12} = \left(\frac{f_1' f_2'}{g}\right)^2. \tag{2.4}$$

The quantities a, b, c, d, α_0, α_1, Δ_H, Δ^α, R^α, T^1, T^2, and F^α are certain functions of the coordinates and the gas-dynamics parameters of the problem and are determined by means of the expressions given in [1], in which the following changes connected with the three-dimensionality of the problem must be introduced:

$$S^{\alpha} = -\frac{f_{\alpha}'^{2} f_{\beta}'^{2}}{g_{(\alpha\alpha)} g^{2}} f_{(\alpha\alpha)}'' + \frac{f_{\alpha}' f_{\beta}' f_{12}'' \left(1 + f_{\alpha}'^{2} - f_{\beta}'^{2}\right)}{g_{(\alpha\alpha)} g^{2}} + \frac{f_{\beta}'^{2} f_{\beta\beta}''}{g^{2}} +$$
$$+ \frac{1 + 2f_{\alpha}'^{2}}{g_{(\alpha\alpha)} g} \left(f_{(\alpha\alpha)}'' + \frac{f_{\beta}'}{f_{\alpha}'} f_{12}'' \right) - 2\lambda g, \quad \alpha \neq \beta,$$

$$h_{3}^{\alpha} = \frac{g}{3r\beta} (k_{2\alpha} - g k_{1\alpha} \lambda), \quad h_{2}^{\alpha} = h_{3}^{\alpha} (b - d), \quad h_{1}^{\alpha} = h_{3}^{\alpha} (b - d)^{2},$$

$$h_{0}^{\alpha} = \frac{2}{a} k_{1\alpha} - h_{3}^{\alpha} t,$$

$$k_{1\alpha} = \frac{g_{\beta\beta}}{f_{\alpha}'} \frac{\partial}{\partial x^{\alpha}} \left(\frac{1}{g} \right) - f_{\beta}' \frac{\partial}{\partial x^{\beta}} \left(\frac{1}{g} \right), \quad k_{2\alpha} = \frac{g_{\beta\beta}}{f_{\alpha}'} \frac{\partial \lambda}{\partial x^{\alpha}} - f_{\beta}' \frac{\partial \lambda}{\partial x^{\beta}}, \quad \alpha \neq \beta,$$

$$b_{3} = 1 - \frac{1 - \mathrm{Pr}}{1 - G_{w}} \varkappa_{\alpha\beta} u_{s}^{\alpha} u_{s}^{\beta}, \qquad q_{3} = \left(\frac{1}{g} - 1 \right) a^{2}, \quad \beta = \frac{2H}{\sqrt{g}},$$

$$H = \frac{1}{2g^{3/2}} \left[f_{11}'' (1 + f_{2}'^{2}) + f_{22}'' (1 + f_{1}'^{2}) - 2 f_{12}'' f_{1}' f_{2}' \right].$$

Here, H is the mean curvature of the surface, equal to the half-sum of the principal curvatures at the point under consideration.

Calculations based on the above expressions and also the numerical solution of the system of equations (1.1) have shown that, for Re ≥ 100, the slippage conditions at the solid and at the shock-wave hardly affect the solution of the problem. If we apply the usual Rankine–Hugoniot conditions and the sticking conditions at the solid, then a = c = 1 and b = d = 0, which considerably simplifies the above expressions.

For instance, we obtain the following expression for the Stanton number:

$$c_{H} = \frac{\cos^{3/2} \alpha \, (2H)^{1/2}}{6 \, (\mathrm{RePr}\, \varepsilon)^{1/2} \, (1 - T_{w})^{1/4} \, (2/27 + (1/7)\, T_{w})^{1/2}}, \tag{2.5}$$

where α is the angle between the vector of the normal to the surface and the vector of the oncoming flow velocity. In our coordinate system, $\cos\alpha = 1/\sqrt{g}$.

The surface distribution of the thermal flux, reduced to its value at the critical point, $q/q_0 = c_H/c_{H0}$ (the subscript 0 pertains to the corresponding quantities at the stagnation point), is found by means of the expression

$$q/q_{0} = \cos^{3/2} \alpha \sqrt{H/H_{0}}, \tag{2.6}$$

which is a generalization of the relationships obtained in [1] for the distribution q/q_0 along the spreading line to the case of the side surface. It follows from (2.6) that, for Re ≥ 100, the relative thermal flux at the side surface no longer depends on Re (for small Re values, this dependence is considerable), while it also is independent of γ, Pr, and T_w (for a cooled wall, $T_w \le 0.5$) and is determined only by the geometric characteristics of the streamlined solid. This is also supported by the results of our numerical calculations.

3. The accuracy of the expressions derived was estimated by comparing them with the numerical solution of the system of equations (1.1) for boundary conditions (1.2) and (1.3). We used the methods [7] of the fourth order of approximation accuracy with respect to the transverse coordinates and of the second order of accuracy with respect to the longitudinal coordinates. The longitudinal components of the pressure gradient were assigned here by using Newton's formula. In order to match the numerical and the analytical solutions, the latter was modified so that p_2^{α} was determined according to Newton's formula. In performing calculations based on (2.1)-(2.4), it was assumed that $k_{2\alpha} = 0$.

The calculation results obtained by means of (2.1)-(2.6) were compared with the exact numerical solutions for different elliptical paraboloids, parted hyperboloids, and triaxial ellipsoids, positioned at a zero angle of attack in the flow. The comparison was carried out in a wide range of gas-dynamics parameters pertinent to the problem: Re = 1-10^4, T_w = 0.01-0.5, and γ = 1.1-1.67. This comparison between the analytical and the numerical solutions has shown that expressions (2.3)-(2.5) ensure satisfactory accuracy at small Reynolds

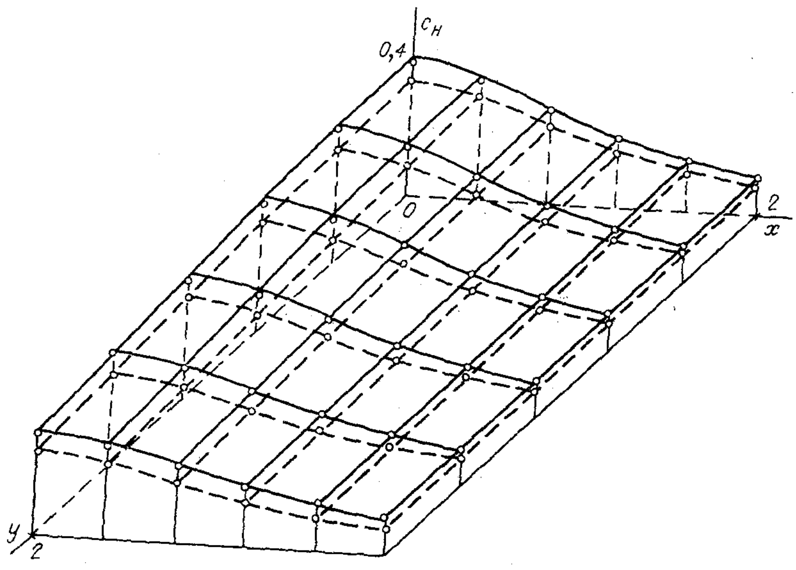

Fig. 1

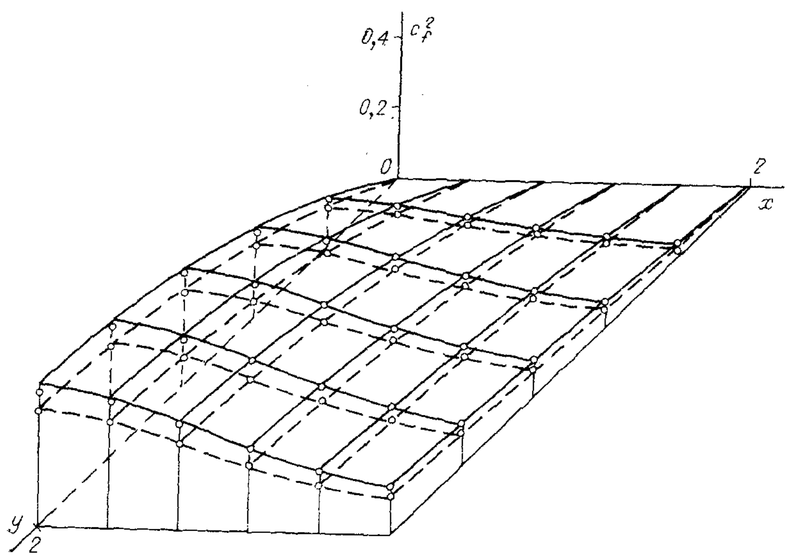

Fig. 2

numbers of the absolute values of the friction and heat exchange coefficients at the surface both with and without an allowance for the slippage effect. Thus, for $Re \leq 10^2$, the error did not exceed 10%. The accuracy of the expressions for the absolute values of c_H and c_f^α diminishes with an increase in Re. Thus, for $Re = 10^3$, the error may reach ~20%, while, for $Re = 10^4$, it can even reach 30%. This is connected with the fact that, in using the method of successive approximations, linear profiles of the velocity and enthalpy components were assigned as the zero approximation, which, for large Re values, no longer corresponded to the behavior of these profiles, even in terms of Dorodnitsyn's variables. However, for these values of Re, we can use the expressions proposed in the boundary layer theory (for instance, [8, 9]).

At the same time, the relative values of the thermal flux and friction stress can be determined with satisfactory accuracy by means of the above expressions throughout the entire range of Re values, from the small to the large ones.

Figures 1 and 2 show the distributions of c_H and of the friction coefficient c_f^2 over the surface of an elliptic paraboloid characterized by the ratio of the principal curvature at the critical point $k = 0.4$ (the results for c_f^1 are similar) for $Re = 1$, $\gamma = 1.4$, $T_w = 0.1$, and $Pr = 0.71$. The solid curves represent the results of calculations of the system of

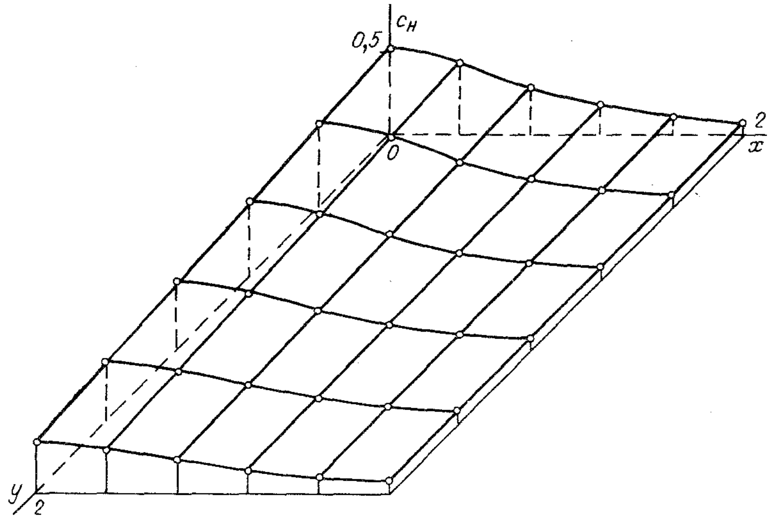

Fig. 3

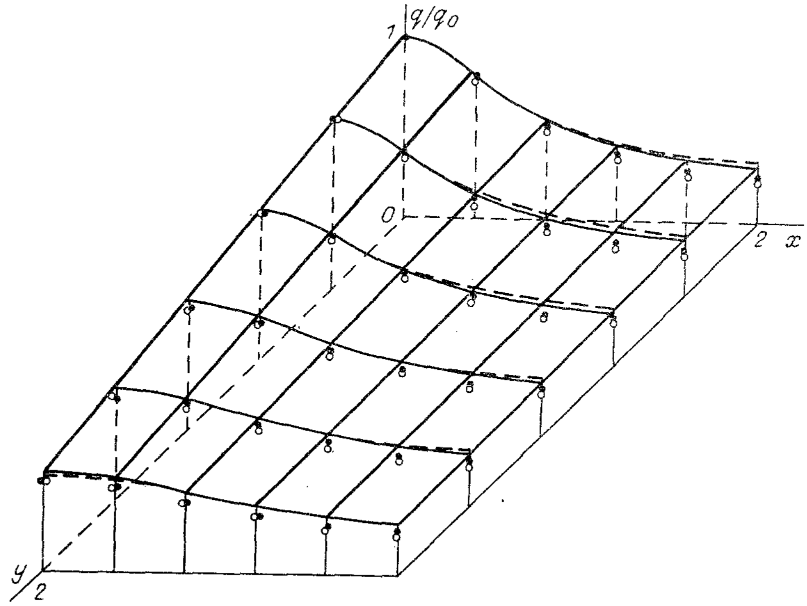

Fig. 4

equations (1.1) without an allowance for the slippage effect, the dashed curves pertain to the results for boundary conditions (1.2), and the circles pertain to calculations based on (2.3) and (2.4). Figure 3 shows the distribution of c_H for the same parameters and Re = 10 with an allowance for the slippage effect, obtained from the numerical solution (solid curves) and the analytical solution (circles).

The distribution of the relative thermal flux over the side surfaces of various solids for medium and large Re numbers is shown in Figs. 4-6, where the dashed curves represent the numerical solution of system (1.1) for Re = 10^2, the solid curves pertain to the solutions for Re = 10^3 and 10^4 (they coincide), the white points refer to calculations based on (2.4) for Re = 10^2, and the black points pertain to the solution based on (2.6). Figures 4 and 5 correspond to the flow around a parted hyperboloid, characterized by a half-angle of 40° in the y = 0 plane and k = 0.5, and a triaxial ellipsoid, characterized by the 1:1.5: 2 ratio between the axes. Figure 6 shows the behavior of q/q_0 as a function of $r = \sqrt{x^2 + y^2}$ over the surface of an elliptic paraboloid with k = 0.4 for differential meridional sections: The curves 1-5 correspond to φ = 0, 45, 63.4, 76, 90° (φ is the angle between the plane of the section in question and the y = 0 plane).

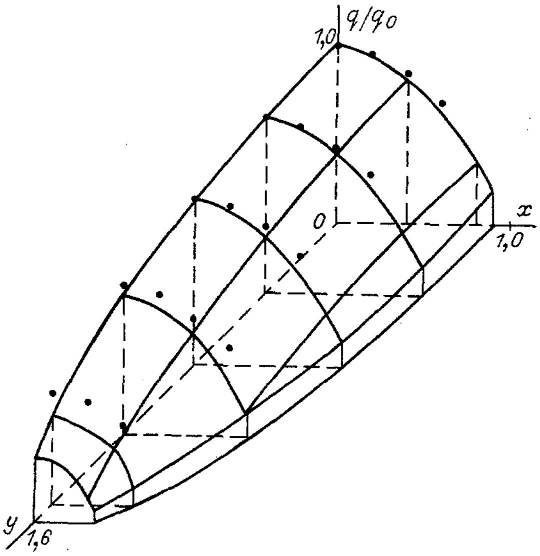

Fig. 5

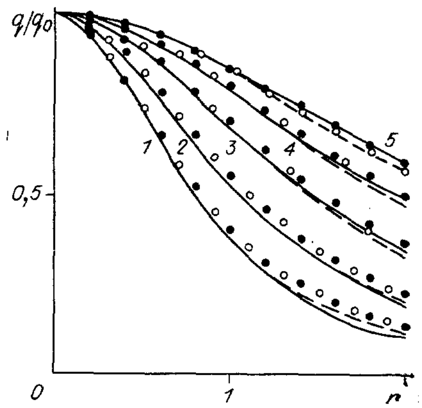

Fig. 6

The above results indicate that for Re ≥ 100, the dependence of the relative thermal flux over the side surface on the Re value virtually vanishes. The calculations performed for the elliptic paraboloid and the hyperboloid have also shown that the value of q/q_0 varies by not more than 5-7% over the side surfaces of these solids with the following parameter variations: $T_w = 0.01\text{-}0.25$ and $\gamma = 1.5\text{-}1.667$. The fact that the relative thermal flux displays a weak dependence on γ and T_w was noted in the earlier investigations, performed within the framework of the boundary layer theory.

Thus, our results indicate that, for Re ≥ 100, the distribution of the relative thermal flux over the surface depends slightly on the gas-dynamics flow parameters Re, γ, T_w, and Pr and is basically determined by the geometric characteristics of the solid. It is described satisfactorily by expression (2.6).

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